

Dec.-22-1353

MA-101L (Applied Mathematics-I)

B.Tech. 1st (CBCS)

Time : 3 Hours

Max. Marks : 60

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt Five questions in all, selecting one question each from Section A, B, C and D. Section E is compulsory.

SECTION - A

1. (a) Prove that the sequence:

$$\left\{ \frac{2n-7}{3n+2} \right\}$$

is (i) monotonically increasing (ii) bounded and (iii) convergent. (5)

(b) Show that the alternating series:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \text{ is convergent. (5)}$$

2. (a) Find the radius of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{(n-1)}{n^n} x^n \quad (5)$$

(b) Show that the function represented by:

$$\sum_{n=1}^{\infty} \frac{\sin nx}{n^3} \text{ is differentiable for every } x \text{ and its derivative is}$$

$$\sum_{n=1}^{\infty} \frac{\cos nx}{n^2} \quad (5)$$

SECTION - B

3. (a) State and prove Cauchy's Mean Value Theorem. (5)

(b) By using Lagrange's Mean Value Theorem, prove that, $|\sin x - \sin y| \leq |x - y|$ for all $x, y \in \mathbb{R}$. (5)

4. (a) Find the area bounded by the parabola $y=2x-x^2$ and the x-axis. (5)

(b) Find the volume of the solid generated by revolving the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

about the x-axis. (5)

SECTION - C

5. (a) Discuss the continuity of $f(x, y)$ at $(0, 0)$ where

$$f(x, y) = \begin{cases} \frac{2xy^2}{x^3 + y^3}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases} \quad (5)$$

(b) Find all the maxima and minima of the function:

$$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy \quad (5)$$

6. (a) If $u = \sin^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$, show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u \quad (5)$$

(b) If $x = r \cos \theta, y = r \sin \theta$, prove that

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0 \text{ except when } x = 0, y = 0. \quad (5)$$

SECTION - D

7. (a) Evaluate :

$$\int_1^2 \int_0^3 (x^2 + y^2) dx dy. \quad (5)$$

- (b) Change the order of integration and evaluate the integral:

$$\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx. \quad (5)$$

8. (a) Evaluate:

$$\iiint (x + y + z) dx dy dz \text{ over the tetrahedron bounded by the planes } x=0, y=0, z=0, x+y+z=1. \quad (5)$$

- (b) Find the area enclosed by the parabolas
- $y^2 = 4ax$
- and
- $x^2 = 4ay$
- ,
- $a > 0$
- by using the double integration.
- (5)

SECTION - E (Compulsory)

9. Attempt all the questions:

- (a) Define convergent sequence.
- (b) State Cauchy's Second Theorem on Limits.
- (c) Define absolutely convergent series.
- (d) Find the value of c of the Lagrange's mean value theorem, if $f(x) = x(x-1)(x-2)$ in $\left[0, \frac{1}{2}\right]$.
- (e) Define rectifiable curve.
- (f) Prove that :

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2 + y^2}} = 0.$$

- (g) State Euler's theorem for a homogeneous function of three variables.
- (h) What are the necessary conditions for maxima and minima of $f(x,y)$?
- (i) Find the area of the circle using the double integration.
- (j) State Cauchy's Root Test. $(10 \times 2 = 20)$